Serb. Astron. J. – OnLine-First Original Scientific Paper

CORRELATION FUNCTION AND ELECTRONIC SPECTRAL LINE BROADENING IN RELATIVISTIC PLASMAS

S. Douis and M.T. Meftah

Department of Physics, Faculty of Sciences,LRPPS Laboratory, University of Kasdi Merbah Ouargla, 30000, Algeria
E–mail: *mewalid@yahoo.com*

(Received: November 9, 2012; Accepted: February 18, 2013)

SUMMARY: The electrons dynamics and the time autocorrelation function $C_{EE}(t)$ for the total electric microfield of the electrons on positive charge impurity embedded in a plasma are considered when the relativistic dynamic of the electrons is taken into account. We have, at first, built the effective potential governing the electrons dynamics. This potential obeys a nonlinear integral equation that we have solved numerically. Regarding the electron broadening of the line in plasma, we have found that when the plasma parameters change, the amplitude of the collision operator changes in the same way as the time integral of $C_{EE}(t)$. The electron-impurity interaction is taken at first time as screened Deutsh interaction and at the second time as Kelbg interaction. Comparisons of all interesting quantities are made with respect to the previous interactions as well as between classical and relativistic dynamics of electrons.

Key words. Relativistic electron trajectory, microfield autocorrelation function, collision operator.

1. INTRODUCTION

The nonlinear behavior of electric charges around an impurity charge of the same sign is a problem that has been studied for a long time due to its great importance in many techniques (Holtsmark 1919, Hooper 1966, Iglesias et al. 1983, Boerker et al. 1987, Berkovsky et al. 1996). Worth to mention that for fully ionized plasma composed of electrons and positive ions, the hypothesis of one component plasma (OCP) allows us to ignore the effects of ions movements to those of electrons because the ratio mass is about $m_e/m_i \approx 1/2000$. So the system is only composed of one kind of mobile charges (electrons), whereas the species of opposite charge (ions) is modeled by the continuous background which provides electrical neutrality. Coulomb forces between point charges are purely repulsive and do not approach very close to each other only rarely whatever the plasma conditions. Concerning the ion-electron interaction, it is clear that it requires a quantum mechanical description. In this case the Coulomb potential is replaced by the finite and regularized potential at the origin (Deutsh 1977, Deutsh et al. 1978, Minoo et al. 1981) $V_{ie}^{SD}(r)(Screened-Deutsh) = -(Ze^2/r)(1-exp(-r/\lambda_T))exp(-r/\lambda_D)$ or Kelbg interaction (Filinov et al. 2003) $V_{ie}^{K}(r) = -\frac{Ze^2}{r\sqrt{\pi}}(1-exp(-r^2/\lambda_T^2)+\sqrt{\pi}\frac{r}{\lambda_T}(1-erf(\frac{r}{\lambda_T})))$ where $\lambda_D = (k_BT/(4\pi n_e e^2))^{1/2}$, $\lambda_T = (2\pi\hbar^2/(m_e k_B T))^{1/2}$ and n_e the density of the electrons, and in this way, the quantum effects are approximately taken into account. Furthermore we will also consider these two potentials for the interaction between the electron and the continuous positive background. We note that many recent works on to the statistical properties of the electrons in the plasmas exist. For example we find in (Dufour et al. 2005, Dufty et al. 2003) that the electrons are considered as classical particles moving with respect to the first Newton law ($d\overrightarrow{v}/dt = -m_e^{-1}\overrightarrow{\nabla}.V(r)$ where m_e is the rest mass of the electron). In addition, the interaction between the electron and the continuous background is taken as purely Coulombic. In our work, we consider the relativistic movement of the electron around the impurity for compute $C_{EE}(t)$. This task passes through two steps: - first step: the computation of the effective potential V(r) in which the electron moves, - second step: we solve the relativistic movement equation for the electron in the effective potential ($d\overrightarrow{p}/dt = -\overrightarrow{\nabla}.V(r)$ where $\overrightarrow{p} = m\overrightarrow{v}$ and $m = m_e/(1 - v^2/c^2)^{1/2}$). Furthermore, when we compute the effective potential V(r), we consider that the electron interacts with the test charge and with the continuous background positive charge via a regularized potential, whereas it interacts with the electrons via Debye potential. In section 2 we construct the equation governing the effective potential on which is centered all the subsequent results of this paper. We also solve this equation and present some discussions about its solutions. The dynamics properties, that is to say, the time autocorrelation function of the electron microfield is presented in section 3. The section 4 applies the previous results of section 3 to the electronic broadening of the line shape in plasmas. At the end we close this paper by a conclusion. Before starting the second section, recall the relevant parameters for our study: the charge number Z, the average distance between electrons $a = (3/4\pi n_e)^{1/3}$, the electron coupling constant $\Gamma = e^2/(kTa)$, the electron Louis de Broglie thermal length λ_T, the degree of quanticity $\eta = \lambda_T/a$ and the Debye length λ_D. The cases reviewed in this work are (for Z=2, Z=4 and Z=8): the coupling parameter $\Gamma = 0.1$, the quanticity parameter $\eta = 0.177$, the dimensionless Debye length $\eta' = \lambda_D/a = 1.826$. These parameters correspond to the electron density $n_e = 2. \times 10^{20} cm^{-3}$ and to the temperature $T = 1.6 \times 10^5 K$. In this region of temperature and electron density, the non-relativistic treatment of the plasma becomes non valid (Mihajlov et al. 2011). We shall then treat the electron movement around the impurity in the framework of the relativistic classical mechanics.

2. INTEGRAL EQUATION FOR THE EFFECTIVE ENERGY POTENTIAL

2.1. Construction of integral equation for the effective energy potential

Let us consider a medium consisting of electrons and a continuous background of neutralizing positive electrical charges. This is the so called model of the one component plasma (OCP). At first, the distribution of the electrons is that of Maxwell-Boltzmann governing the equilibrium state of the electrons' system. If we place a positive ion of charge Ze (called test charge or impurity) at the coordinates origin: the system is disturbed and after a certain time t, it will reach a new equilibrium state described by a novel distribution of the electrons over the space around the charge Ze. The latter is determined through the potential energy of an electron located at a distance r from the test charge Ze when the system reaches this novel equilibrium state. This potential energy is built as a sum of three contributions:

$$V(r) = V_{ie}(r) + V_{ee}(r) + V_{ef}(r) \tag{1}$$

where $V_{ie}(r)$ is the potential energy of ion-electron interaction (the ion is the test charge), $V_{ee}(r)$ is the interaction energy of the electron with all the other electrons and $V_{ef}(r)$ is the interaction energy of the electron with the continuous neutralizing background of ions (Kalman et al. 2002,Talin et al. 2002). The ion-electron interaction is taken in a way that we can consider the quantum effects at short distances: we represent it here by the following pseudo-potential (Deutsh 1977, Deutsh et al. 1978, Minoo et al. 1981):

$$V_{ie}^{SD}(r) = -\frac{Ze^2}{r}(1 - e^{-r/\lambda_T})e^{-r/\lambda_D} \tag{2}$$

or Kelbg interaction:

$$V_{ie}^{K}(r) = -\frac{Ze^2}{r\sqrt{\pi}}.(1 - e^{-\frac{r^2}{\lambda_T^2}}) + \frac{r\sqrt{\pi}}{\lambda_T}(1 - erf(\frac{r}{\lambda_T}))) \tag{3}$$

We will first investigate the case of screened Deutsh potential and the results for Kelbg potential are straightforward. Most previous studies (Talin et al. 2002, Dufty et al. 2003, Dufour et al. 2005) taken the Coulomb interaction like electrons interactions with themselves and with uniform neutralizing background of positive electric charge. For approach to the reality and taking into account the effect of screening in our study we will take the electron-electron interaction is that of the Debye potential energy $e^2 e^{-r/\lambda_D}/r$ such as the potential energy $V_{ee}(r)$, in the mean field approximation is equal to:

$$V_{ee}(r) = e^2 \int f(\overrightarrow{r'}, \overrightarrow{p}) \frac{e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_D}}}{|\overrightarrow{r} - \overrightarrow{r'}|} d\overrightarrow{p}^3 d\overrightarrow{r'}^3 \tag{4}$$

where:

$$f(\overrightarrow{r}, \overrightarrow{p}) = \frac{N}{\Omega}(\frac{m\beta}{2\pi})^{3/2} e^{-\beta(\frac{\overrightarrow{p}^2}{2m} + V(r))} \tag{5}$$

is the Maxwell-Boltzmann distribution and N is the total number of electrons and Ω is the volume of the system, whereas the energy potential of the electron with the positive background neutralizing charge is given by:

$$V_{ef}^{SD}(r) = -n_e e^2 \int \frac{(1 - e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_T}})e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_D}}}{|\overrightarrow{r} - \overrightarrow{r'}|} d\overrightarrow{r'}^3 \tag{6}$$

In this formula, we have introduced screened Deutsh interaction between the electron and the continuous background of positive charge. Then the potential interaction energy of the electron, with all the plasma components, satisfies the following nonlinear integral equation:

$$\begin{aligned} V^{SD}(r) &= V_{ie}^{SD}(r) + \\ &e^2 \int\int f(\overrightarrow{r'}, \overrightarrow{p}) \frac{e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_D}}}{|\overrightarrow{r} - \overrightarrow{r'}|} d\overrightarrow{p}^3 d\overrightarrow{r'}^3 \\ &-n_e e^2 \int \frac{(1 - e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_T}})e^{-\frac{|\overrightarrow{r} - \overrightarrow{r'}|}{\lambda_D}}}{|\overrightarrow{r} - \overrightarrow{r'}|} d\overrightarrow{r'}^3 \end{aligned} \tag{7}$$

Using spherical coordinates and some basic calculations, the last integral equation is transformed into the following:

$$\begin{aligned} V^{SD}(r) &= V_{ie}^{SD}(r) + 2\pi n_e e^2 \int_0^\infty \frac{r' dr'}{r} \lambda_D. \\ &(e^{-(r+r')/\lambda_D} - e^{-|r-r'|/\lambda_D})(1 - e^{-\beta V^{SD}(r')}) \\ &-2\pi n_e e^2 \int_0^\infty \frac{r' dr'}{r} \lambda (e^{-\frac{r+r'}{\lambda}} - e^{-\frac{|r-r'|}{\lambda}}) \end{aligned} \tag{8}$$

where $\lambda = \lambda_D \lambda_T / (\lambda_T + \lambda_D)$.

In order to deal with an adimensional equation, put $Y^{SD}(x) = -aV^{SD}(r)/(Ze^2)$, $a = (4\pi n_e/3)^{-1/3}$, $x = r/a$, $\eta = \lambda_T/a$, $\eta' = \lambda_D/a$ and $\xi = \eta\eta'/(\eta + \eta')$. After that, we obtain the desired adimensional integral equation:

$$\begin{aligned} Y^{SD}(x) &= \frac{1}{x}(1 - e^{-x/\eta})e^{-x/\eta'} \\ &-\frac{3}{2Z}\int_0^\infty \frac{t}{x}[\eta'(e^{-\frac{x+t}{\eta'}} - e^{-\frac{|x-t|}{\eta'}})(1 - e^{Z\Gamma Y^{SD}(t)}) \\ &-\xi(e^{-\frac{x+t}{\xi}} - e^{-\frac{|x-t|}{\xi}})]dt \end{aligned} \tag{9}$$

The same calculations for the case of Kelbg interaction give:

$$\begin{aligned} Y^K(x) &= Y_{ie}^K(x) + \\ &\frac{3}{2Z}\int_0^\infty \frac{t}{x}[\eta'(e^{-\frac{x+t}{\eta'}} - e^{-\frac{|x-t|}{\eta'}})e^{Z\Gamma Y^K(t)} \\ &-\frac{1}{\eta\sqrt{\pi}}(F(t+x) - F(|x-t|))]dt. \end{aligned} \tag{10}$$

where:

$$Y_{ie}^K(x) = \frac{1}{x\sqrt{\pi}}[1 - e^{-(\frac{x}{\eta})^2} + \frac{x\sqrt{\pi}}{\eta}(1 - erf(\frac{x}{\eta}))] \tag{11}$$

and

$$\begin{aligned} F(x) &= -x(\eta + x\frac{\sqrt{\pi}}{2} + \frac{\eta}{2}e^{-(\frac{x}{\eta})^2}) + \\ &\eta^2\frac{\sqrt{\pi}}{2}erf(\frac{x}{\eta})(\frac{3}{2} + (\frac{x}{\eta})^2) \end{aligned} \tag{12}$$

It should be noted here that (Shukla et al. 2008) has also studied the electron dynamics around an impurity, by considering the hot and degenerate electrons. For this (Shukla et al. 2008), the quantum distribution of Thomas-Fermi was used.

2.2. Numerical solution of the integral equation for the potential energy

We can solve the nonlinear integral equation (9) by the method of successive iterations (fixed point method FPM) starting with the initial function $Y_0(x) = Y_{ie}^{SD}(x) = \frac{1}{x}(1 - e^{-x/\eta})e^{-x/\eta'}$. We can also solve this integral equation by transforming it into a second order nonlinear differential equation and then we use the method of Runge-Kutta (RKM) to solve it. The numerical solution of the nonlinear integral equations (9-10), in the case $\Gamma = 0.1$, $\eta = 0.177$, $\eta' = 1.826$ and $Z = 2$ and $Z = 8$ by the iterative method gives the potential energy as shown in figrs.1-2.

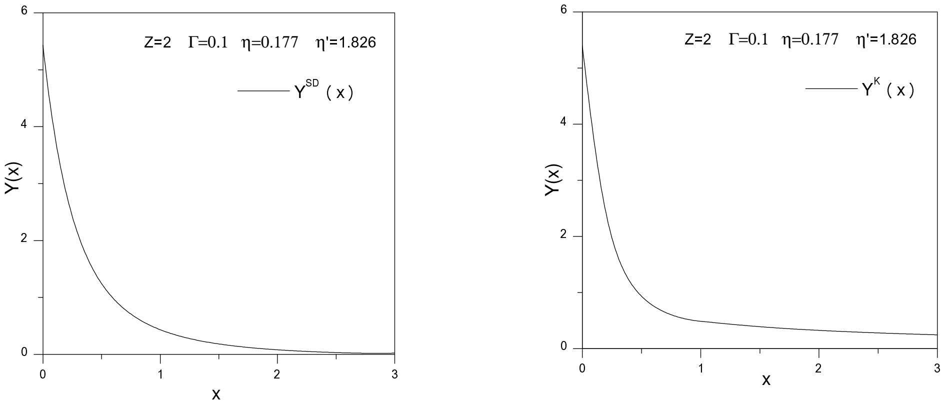

Fig. 1. *Effective potential energy of the electron for Z=2 in Deutsh and Kelbg cases.*

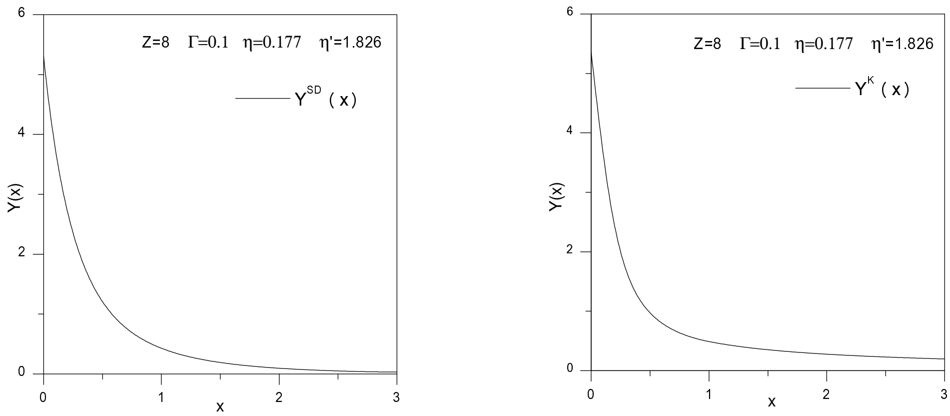

Fig. 2. *Effective potential energy of the electron for Z=8 in Deutsh and Kelbg cases.*

On figrs.1-2, we notice that using Kelbg potential as initial potential and interaction potential between the electron and the continuous background of positive charge, in the integral equation, gives a solution decreasing much weaker than obtained when using screened Deutsh potential. This means that Deutsh solution is strongly screened, that is to say that its range is too inferior to that of Kelbg solution. The RKM allows us to solve, equivalently to the integral equation, the nonlinear differential equation for the Deutsh case. By this way we have solved the equation:

$$Y'' + \frac{2}{r}Y' = \frac{3}{Z}(e^{Z\Gamma.Y(r)} - 1 + (\frac{\xi}{\eta'})^2) + \frac{1}{\eta'^2}Y(r) - \frac{1}{r\eta}(\frac{2}{\eta'} + \frac{1}{\eta})e^{-r(\frac{1}{\eta'}+\frac{1}{\eta})} \quad (13)$$

with initial conditions:

$$Y(0) \approx 1/\eta \;\; and \;\; Y'(0) \approx -(\eta'/2\eta + 1)/\eta\eta' \quad (14)$$

Fig.3 shows this equivalence, for Deutsh case, on the effective potential $Y^{SD}(r)$ when Z=8, $\Gamma = 0.1$, $\eta = 0.177$, $\eta' = 1.826$.

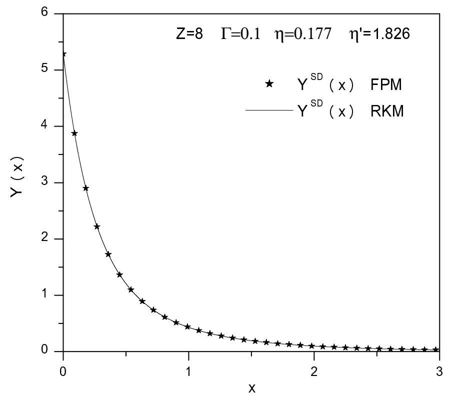

Fig. 3. *Effective potential energy of the electron for Z=8 in Deutsh case computed with Fixed Point Method(* * *) and Runge-Kutta Method (—).*

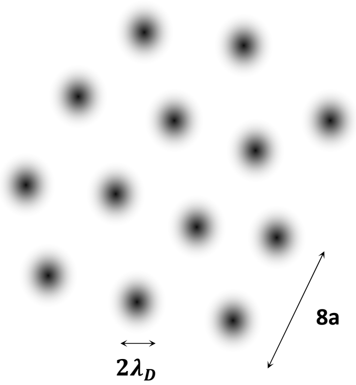

Fig. 4. *Schematic map of electrons distribution around the impurities in the plasma.*

We have also found that the RKM (differential equation) is faster than the fixed point method (integral equation). The drawback of the RKM is very sensitive to the initial conditions. Conversely, the FPM, despite that it requires much time for computation, it has more guarantees that the result converges towards an exact solution. Another drawback of the FPM is that adaptable only for the shielded initial potential.

Mathematically the integral equation (9) admits finite solutions at short distance (when r have approximately the screening length as shown in fig.4). This rapid convergence towards the solution is guaranteed by the screening effect. Physically, this may be interpreted by the following reasoning: a non bounded electron interacts with a neighborhood of some mean inter-electron distance a. This neighborhood contains electrons that are distributed with a density $n_e(r)$ around a single impurity of positive electric charge. This means that the impurities are distributed in plasma with a mean constant density such that the neighborhood of each electron contains only a single impurity (see schematic map in fig.4). What has just been said suggests that numerical integration of the integral equation or the equation differential must be truncated to the size of this neighborhood. When the screening is weak (that is to say that λ_D is very large) the convergence towards the solution of the integral equation (9) becomes slow and the solution coincides at large distance with kelbg solution, see figrs 1-2.

3. THE DYNAMICAL PROPERTIES OF THE ELECTRONS

3.1. Relativistic electron trajectories in plasma

The calculation of the real trajectories of relativistic electrons in a hot plasma, is a necessary step to calculate several dynamic properties such as the time autocorrelation function, the diffusion coefficient and the electric permittivity... So the purpose of this section is the calculation of relativistic trajectories of an electron in a plasma, and then we compare between classical and relativistic trajectories for few cases of potential energy. The relativistic force acting on an electron in the plasma is equal to the derivative of the momentum $\overrightarrow{P}$:

$$\overrightarrow{F} = \frac{d\overrightarrow{P}}{dt} = \frac{dm}{dt}\overrightarrow{v} + m\frac{d\overrightarrow{v}}{dt} \tag{15}$$

and the derivative of the mass is:

$$\begin{aligned} \tfrac{dm}{dt} &= m_e \tfrac{d}{dt}\left(1/\sqrt{1-v^2/c^2}\right) \\ &= m_e \tfrac{1}{c^2}\overrightarrow{v}\cdot\overrightarrow{\gamma}\left(1-v^2/c^2\right)^{-3/2} \end{aligned} \tag{16}$$

where $\overrightarrow{\gamma}$ is the relativistic acceleration, m_e is the rest mass of the electron and c the speed of light in vacuum. Therefore this force is equal to:

$$\overrightarrow{F} = \frac{m_e}{c^2}\left(1-v^2/c^2\right)^{-3/2}\left(\overrightarrow{v}\cdot\overrightarrow{\gamma}\right)\overrightarrow{v} + \frac{m_e}{\sqrt{1-v^2/c^2}}\overrightarrow{\gamma} \tag{17}$$

and the force on the other hand equal:

$$\overrightarrow{F} = \frac{d\overrightarrow{P}}{dt} = -\overrightarrow{\nabla}V(r) \tag{18}$$

where $V(r)$ represents the potential energy (8) of an electron plasma at position r from the coordinates origin. We write the equations (17) and (18) in cartesian coordinates, and we equate member to member, we find the following system of equations:

$$\begin{cases} \frac{m_e\omega^3}{c^2}v_x\left((v_x+\frac{c^2}{\omega^2 v_x})\gamma_x+v_y\gamma_y+v_z\gamma_z\right)=-\frac{x}{r}\frac{\partial V(r)}{\partial r} \\ \frac{m_e\omega^3}{c^2}v_y\left(v_x\gamma_x+(v_y+\frac{c^2}{\omega^2 v_y})\gamma_y+v_z\gamma_z\right)=-\frac{y}{r}\frac{\partial V(r)}{\partial r} \\ \frac{m_e\omega^3}{c^2}v_z\left(v_x\gamma_x+v_y\gamma_y+(v_z+\frac{c^2}{\omega^2 v_z})\gamma_z\right)=-\frac{z}{r}\frac{\partial V(r)}{\partial r} \end{cases} \quad (19)$$

where: $\omega = 1/\sqrt{1-\left(v_x^2+v_y^2+v_z^2\right)/c^2} = 1/\sqrt{1-v^2/c^2}$

Solving this system of equations gives the expression of the acceleration as follows:

$$\overrightarrow{\gamma}: \begin{cases} \gamma_x=\zeta\left[+\left(c^2-v_x^2\right)x-yv_xv_y-zv_xv_z\right] \\ \gamma_y=\zeta\left[-xv_xv_y+\left(c^2-v_y^2\right)y-zv_yv_z\right] \\ \gamma_z=\zeta\left[-xv_xv_z-yv_yv_z+\left(c^2-v_z^2\right)z\right] \end{cases} \quad (20)$$

where: $\zeta=-\frac{1}{m_ec^2\omega}\frac{1}{r}\frac{\partial V(r)}{\partial r}$ and now we can use the following Taylor formula to find the trajectories of relativistic electrons:

$$\overrightarrow{r}(t+\Delta t)=2\overrightarrow{r}(t)-\overrightarrow{r}(t-\Delta t)+(\Delta t)^2\,\overrightarrow{\gamma}(t) \quad (21)$$

In figrs.5-6, we present the difference between classical and relativistic trajectories of an electron plasma governed by the effective potential (9) and (10). These figures show that the movement of electrons in a cold plasma around the impurity center can be bound, but the electrons in the hot plasma are free due to their high velocities and the bounded trajectories does not appear in relativistic movement.

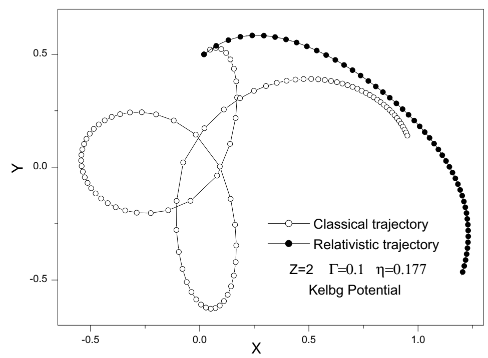

Fig. 5. *classical and relativistic trajectories for the initial conditions: r(0)=0.7 and v(0)=0.485*

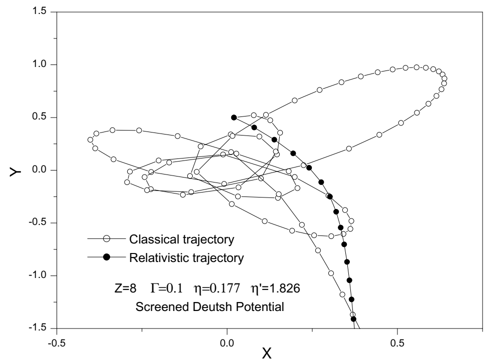

Fig. 6. *classical and relativistic trajectories for the initial conditions: r(0)=0.76 and v(0)=1.04*

3.2. The microfield autocorrelation function.

The total electric microfield due to the electrons on the impurity centered at the coordinates origin is given by:

$$\overrightarrow{E}=\sum_{k=1}^{N}\overrightarrow{e_{ie}}(r_k) \quad (22)$$

then the dimensionless electric field auto-correlation function is given by (Talin et al. 2008):

$$C_{EE}(t)=\frac{a^4}{e^2}\left\langle \overrightarrow{E}(t)\cdot\overrightarrow{E}\right\rangle$$
$$=\frac{a^4}{e^2}\int d\overrightarrow{r}_1 d\overrightarrow{v}_1....d\overrightarrow{r}_N d\overrightarrow{v}_N \overrightarrow{E}\cdot\overrightarrow{E}(-t)\rho_e$$
$$=\frac{a^4}{e^2}\int d\overrightarrow{r}_1 d\overrightarrow{v}_1\overrightarrow{e}(r_1)N$$
$$\int d\overrightarrow{r}_2 d\overrightarrow{v}_2....d\overrightarrow{r}_N d\overrightarrow{v}_N\overrightarrow{E}(-t)\rho_e$$
$$=\frac{a^4}{e^2}\int d\overrightarrow{r}_1 d\overrightarrow{v}_1\overrightarrow{e}(r_1)\Psi(\overrightarrow{r}_1,\overrightarrow{v}_1,t) \quad (23)$$

where ρ_e is the equilibrium canonical ensemble and $e(r_\alpha)$ is the single particle field. The integrations over degrees of freedom $2...N$ in the second equality define a reduced function $\Psi(r_1,v_1,t)$, which is the first member of a set of such functions

$$\Psi(\overrightarrow{r}_1,\overrightarrow{v}_1....\overrightarrow{r}_s,\overrightarrow{v}_s,t)=$$
$$N^s\int d\overrightarrow{r}_{s+1}d\overrightarrow{v}_{s+1}....d\overrightarrow{r}_N d\overrightarrow{v}_N\overrightarrow{E}(-t)\rho_e. \quad (24)$$

It is straightforward to verify that these functions satisfy the BBGKY hierarchy (Van Kampen et al.

1967).

$$(\partial_t + \overrightarrow{v} \cdot \overrightarrow{\nabla}_r - \frac{(1-\frac{v^2}{c^2})^{1/2}}{m_e}\{\overrightarrow{\nabla}_r[V_{ie}(r) + V_{ef}(r)]\}$$
$$\cdot\{\overrightarrow{\nabla}_v - \frac{\overrightarrow{v}}{c}(\frac{\overrightarrow{v}.\overrightarrow{\nabla_v}}{c})\})\Psi(\overrightarrow{r}, \overrightarrow{v}, t)$$
$$= \frac{1}{m_e\omega}\int d\overrightarrow{r}_1 d\overrightarrow{v}_1[\overrightarrow{\nabla}_r V_{ee}(\overrightarrow{r} - \overrightarrow{r}_1)]$$
$$\cdot\{\overrightarrow{\nabla}_v - \frac{\overrightarrow{v}}{c}(\frac{\overrightarrow{v}.\overrightarrow{\nabla_v}}{c})\}\Psi^{(2)}(\overrightarrow{r}, \overrightarrow{v}; \overrightarrow{r}_1, \overrightarrow{v}_1, t) \quad (25)$$

where m_e is the electron mass at the rest and c is the light velocity. Recognizing this linear relationship, the basic approximation for weak coupling among the electrons is to neglect all of their correlations at all times.

$$\Psi^{(2)}(\overrightarrow{r}_1, \overrightarrow{v}_1, \overrightarrow{r}_2, \overrightarrow{v}_2, t) \rightarrow f(\overrightarrow{r}_2, \overrightarrow{v}_2)\Psi(\overrightarrow{r}_1, \overrightarrow{v}_1, t)$$
$$+f(\overrightarrow{r}_1, \overrightarrow{v}_1)\Psi(\overrightarrow{r}_2, \overrightarrow{v}_2, t) \quad (26)$$

Use of (26) in first hierarchy equation (25) gives directly the kinetic equation

$$(\partial_t + L)\,\Psi(\overrightarrow{r}, \overrightarrow{v}, t) =$$
$$\frac{1}{m_e\omega}[\overrightarrow{\nabla_v} - \frac{\overrightarrow{v}}{c^2}(\overrightarrow{v}.\overrightarrow{\nabla_v})]f(\overrightarrow{r}, \overrightarrow{v})\cdot$$
$$\overrightarrow{\nabla}_r \int d\overrightarrow{r}_2 V_{ee}(\overrightarrow{r} - \overrightarrow{r}_2)\int d\overrightarrow{v}_2\Psi(\overrightarrow{r}_2, \overrightarrow{v}_2, t) \quad (27)$$

where

$$L = \overrightarrow{v}.\overrightarrow{\nabla} - \frac{1}{m_e\omega}\left[\overrightarrow{\nabla}_r V(r)\right].(\overrightarrow{\nabla}_v - \frac{\overrightarrow{v}}{c^2}(\overrightarrow{v}.\overrightarrow{\nabla_v}) \quad (28)$$

Limit ourselves to the solution of the homogeneous equation of (27) which is given by:

$$\Psi(\overrightarrow{r}, \overrightarrow{v}, t) = f(\overrightarrow{r}, \overrightarrow{v})\overrightarrow{e}_{mf}(\overrightarrow{r}(t)) \quad (29)$$

where:

$$\overrightarrow{e}_{mf}(\overrightarrow{r}) = \frac{1}{Ze}\overrightarrow{\nabla}V(r) \quad (30)$$

and $f(r, v)$ is the Maxwell-Juttner-Boltzmann distribution given by:

$$f(r, v) = \frac{exp(-(mc^2 + V(r))/kT)}{m_e^3 c^3 K_2(m_e c^2/kT)} \quad (31)$$

here $m = m_e\omega$ and $K_2(x)$ is the modified Bessel function. Replace (29) in (23) we find:

$$C_{EE}(t) = \frac{a^4}{e^2}\int f(\overrightarrow{r}, \overrightarrow{v})\,\overrightarrow{e}(r)\cdot\overrightarrow{e}_{mf}(r(t))d\overrightarrow{r}d\overrightarrow{v} \quad (32)$$

where $\overrightarrow{r}(t)$ is the time-dependent position vector. We get it at all time t, we have solved numerically (using Verlet algorithm) the movement equation $d\overrightarrow{P}/dt = -e.\overrightarrow{e}_{mf}(r)$ with $\overrightarrow{P} = m_e\overrightarrow{v}/\sqrt{(1 - v^2/c^2)}$ is the relativistic momentum of the electron. In the calculation of $C_{EE}(t)$ the average on the velocities is done on the relativistic distribution $f(r, v)$.

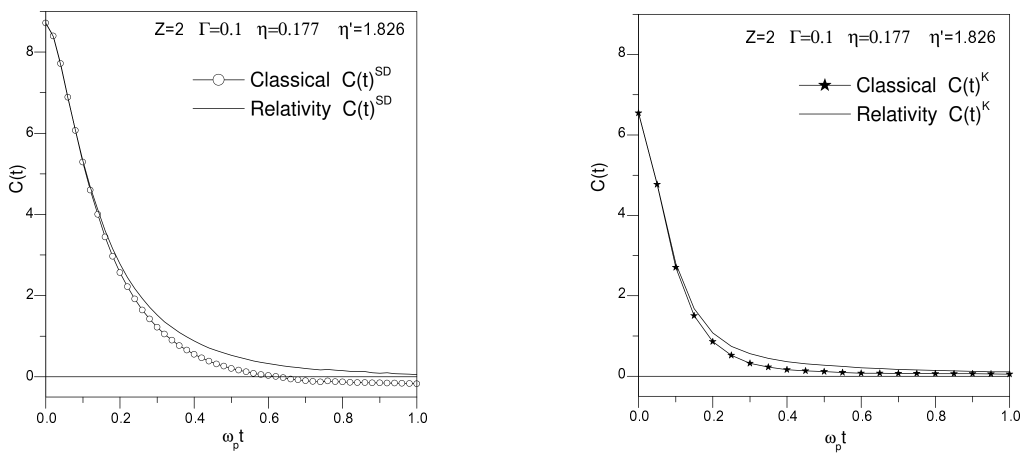

Fig. 7. *Classical and relativistic electric field auto-correlation function for Z=2 in Deutsh and Kelbg cases*

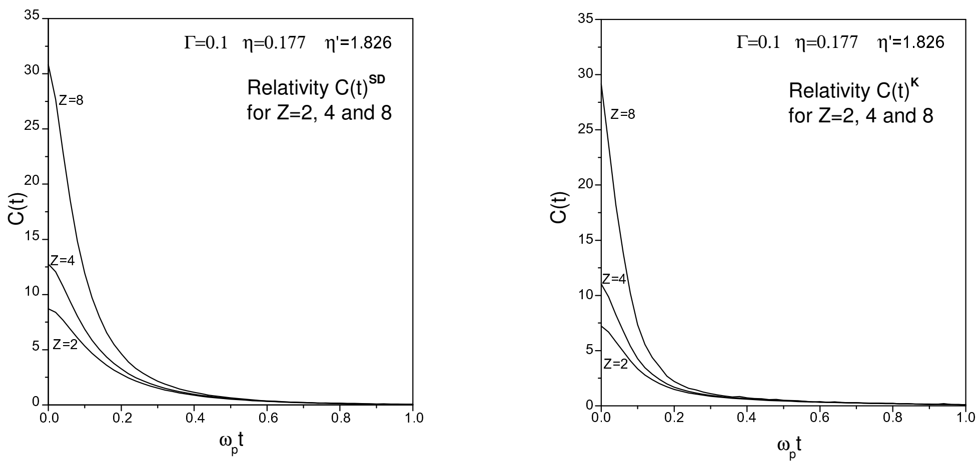

Fig. 8. *Relativistic electric field auto-correlation function for different Z in Deutsh and Kelbg cases*

Regarding the function $C_{EE}(t)$ in Fig.7, we found: - When you move away from $t = 0$, the relativistic effect is manifested more clearly. - In Fig.8, we note that when we increase the charge number Z the covariance $C(0)$ also increases, but all the relativistic curves in Kelbg and Deutsh cases decrease more quickly and have the same behavior at large time (vanished quickly and never cut the Ox axis).

4. APPLICATION TO THE ELECTRONIC BROADENING IN PLASMAS

First we have to keep in mind that (Alexiou 1994) and predecessors GBKO (Griem et al. 1962) considered the interaction between the electrons and the emitter in the impact approximation. Then begin by outlining the process that led (Alexiou 1994, Griem et al. 1962) with hyperbolic trajectories to construct a valid electronic collision operator for isolated lines. They considered the interaction of a plasma electron with an ion is purely Coulombic, and in addition the movement of the plasma electron is due solely to the Coulomb field created by the ion. The field of the electron in an ion emitter is then Coulombic. We now introduce some new notations. First, we use the time-dependent interaction between a single plasma electron and emitter electron as follow:

$$V(t) = \overrightarrow{d}.\overrightarrow{e}(t) = -e\overrightarrow{R}.\overrightarrow{e}(\overrightarrow{r}(t)) \qquad (33)$$

where $\overrightarrow{R}$ is the position vector of the emitter electron and $\overrightarrow{e}(t)$ is the individual electric field on the impurity due to the electron located at $\overrightarrow{r}$. Now we want to separate the purely atomic part and the part that depends on the details of the collision process. This leads to defining two quantities (Alexiou 1994):

$$\phi_d = -2\pi\frac{n_e e^2}{3\hbar^2}\int vF(v)dv\int \rho d\rho\int_{-\infty}^{\infty}dt_1 \int_{-\infty}^{t_1}dt_2 e^{i\omega_1 t_1}e^{i\omega_2 t_2}\overrightarrow{e}(t_1).\overrightarrow{e}(t_2) \qquad (34)$$

and:

$$\phi_{int} = -2\pi\frac{n_e e^2}{3\hbar^2}\int vF(v)dv\int \rho d\rho\int_{-\infty}^{\infty}dt_1 \int_{-\infty}^{\infty}dt_2 e^{i\omega_1 t_1}e^{i\omega_2 t_2}\overrightarrow{e}(t_1).\overrightarrow{e}(t_2) \qquad (35)$$

where $F(v)$ is the Maxwell distribution of the velocities. The integral:

$$2\pi n_e\int vF(v)dv\int \rho d\rho \qquad (36)$$

defines the average in the phases space of the particle positions $\overrightarrow{r}_i$ and the particle momentum $\overrightarrow{p}_i$. The distribution is that of Maxwell-Boltzmann $F(v)exp(-V/kT)$, then we can transform ϕ_d as:

$$\phi_d = -\frac{e^2}{3\hbar^2}\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{t_1}dt_2 e^{i\omega_1 t_1+i\omega_2 t_2} < \overrightarrow{E}(t_1).\overrightarrow{E}(t_2) >_{can} \qquad (37)$$

where $\overrightarrow{E}(t)$ is the total field created by the electrons of the plasma on the emitter. By making use of the stationarity:

$$C_{EE}(t_1 - t_2) =< \overrightarrow{E}(t_1).\overrightarrow{E}(t_2) >_{can} \qquad (38)$$

we can check that:

$$\phi_d = -\frac{e^4}{3\hbar^2 a^4}\int_0^{\infty}C_{EE}(t)dt \qquad (39)$$

(in s^{-1} unit) where $C_{EE}(t)$ is given by (32). The same formula is used by (Nguyen et al. 1967) for the collision operator:

$$\phi = -\overrightarrow{R_n}.\overrightarrow{R_n}(\frac{e^4}{3.\hbar^2 a^4}) \int_0^{+\infty} C_{EE}(t)dt \equiv \overrightarrow{R_n}.\overrightarrow{R_n}\phi_d \tag{40}$$

In computing the collision operator ϕ, conversely to (Alexiou 1994) and his predecessors, the plasma electron (the perturber) moves in the effective field created by all the plasma. Moreover this electron creates a field(Deutsh or Kelbg) at the impurity ion. Then we call ϕ_d the amplitude of the collision operator because it is this quantity that contains the plasma parameters through the correlation function $C_{EE}(t)$. This contains all the information regarding the density n_e, the temperature T and the charge number Z of ions. We present on a table the ratio of the amplitudes of the collision operator between the classical and relativistic case for different plasma conditions such as n_e, T and Z. We find that when Z is increased the relativistic effect increases.

Table 1. The ratio $\phi_d^{classical}/\phi_d^{relativistic}$.

Z	Deutsh	Kelbg
1	0.9052	0.9154
2	0.8102	0.8263
4	0.5767	0.6554
8	0.3256	0.3396

From this table, we see that the difference between the classical and relativistic collision operator becomes more important when the number of charge Z increases.

5. CONCLUSION

The electrons dynamics and the time autocorrelation function $C_{EE}(t)$ for the total electric microfield of the electrons on positive charge impurity embedded in a plasma are considered when the relativistic dynamic of the electrons is taken into account. We have, at first, built the effective potential governing the electrons dynamics. This potential obeys a nonlinear integral equation which we have solved numerically. We have found, for fixed $\Gamma = 0.1$ and fixed density n_e, when the charge number Z increases the relativistic effect becomes important (for $Z = 8$, $\phi_{classical}/\phi_{relativistic} \simeq 1/3$). The collision operator, responsible for electronic broadening in plasma, is investigated: the result is when the plasmas parameters change, the amplitude of the collision operator becomes important. The electron-impurity interaction is taken to be in first time as screened Deutsch interaction and in second time to be Kelbg interaction. Comparisons of all interests quantities are made for these interactions as well as between classical and relativistic dynamics of electrons. The relativistic trajectories of the plasma electrons around the impurity are more complicated from those the classical trajectories as it can be seen in Fig.5-6. This fact has a direct effect on the behavior of the electric auto-correlation function. Indeed, when we move away from $t = 0$, the difference between the classical C_{EE} and the relativistic C_{EE} becomes more net.

Acknowledgements – During the preparation of this paper we were financially supported by the Ministry of Education and Science and Research (PNR project) and LRPPS Laboratory at Ouargla University of the Republic, Democratic and Popular of Algeria for which we present our full and high acknowledgements.

REFERENCES

Holtsmark, J.: 1919, Ann. Phys. **58**, 577.
Hooper, C. F.: 1966, Phys.Rev. Vol.149, Num.1, 77.
Iglesias, C. A., Lebowitz, J. L., Gowan, D. MC.: 1983, Phys. Rev. A **28**, 1667.
Boercker, D. B., Iglesias, C. A., Dufty, J. W.: 1987, Phys. Rev. A **36**, 2254.
Berkovsky, M. A., Dufty, J. W., Calisti, A., Stamm, R., Talin, B.: 1996, Phys. Rev. E **54**, 4087.
Deutsh, C.: 1977, Phys. Lett. A **60**, 317.
Deutsh, C., Gombert, M. M., Minoo, H.: 1978, Phys. Lett. A **66**.
Minoo, H., Gombert, M.M., Deutsh, C.: 1981, Phys. Rev. A **23**, 924.
Filinov, A.F., Bonitz, M., Ebeling, W.: 2003, J. Phys. A **36**, 5957.
Dufour, E., Calisti, A., Talin, B., Gigosos, M., Gonzalez, M., del Ro Gaztelurrutia, T., Dufty, J. W.: 2005, Phys. Rev. E **71**, 066409.
Dufty, J. W., Pogorelov, I., Talin, B., Calisti, A.: 2003, J. Phys. A **36**, 6057.
Mihajlov, A. A., Sakan, N. M., Srećković, V. A., Vitel, Y.: 2011, J. Phys. A: Math. Theor. **44** 095502.
Donkó, Z., Kalman, G. J., Golden, K. I.: 2002, Phys. Rev. Lett. **22**, 88.
Ali, S., Shukla, P. K.: 2008, Phys. Lett. **A**, vol.372, 4827.
Talin, B., Calisti, A., Dufty, J. W., Pogorelov, I. V.: 2008, Phys. Rev. E **77**, 036410.
Van Kampen, N. G., Felderhof, B. U.: 1967, Theoretical Methods in plasma physics (North-Holland Publishing Amsterdam), Chap.8 170.
Alexiou, S.: 1994, Phys. Rev. A **49**, 106.
Griem, H. R., Baranger, M., Kolb, AC., Oertel, G.: 1962, Phys. Rev. 125(1). 177.
Nguyen-Hoe, Drawin, H. W., Herman, L.: 1967, J. Q. S. R. T. 429.